PARAMETER ESTIMATION FOR THE MATHEMATICAL MODELING OF SEMICONDUCTORS USING THE EBERS-MOLL EQUATIONS

by

William Arthur Crumly

United States Naval Postgraduate School

THESIS

PARAMETER ESTIMATION FOR THE MATHEMATICAL MODELING OF SEMICONDUCTORS USING THE EBERS-MOLL EQUATIONS

by

William Arthur Crumly

December 1969

Parameter Estimation for the Mathematical Modeling
of Semiconductors Using the Ebers-Moll Equations

by

William Arthur Crumly
Lieutenant, United States Naval Reserve
B.S., Iowa Wesleyan College, 1963

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1969

ABSTRACT

An accurate method of parameter estimation for the mathematical modeling of semiconductors using the Ebers-Moll equations is presented. Its usefulness is apparent in estimating parameters to be used in computer circuit-analysis programs that have been developed. The Ebers-Moll models were modified to better represent the actual characteristics. The least-square-error methods presented for estimating the parameters are easy to program to the digital computer and result in parameters that are quite accurate in describing the actual characteristics. This procedure yields solutions for parameters that are quite helpful to the engineer in solving electrical circuits involving p-n diodes and junction transistors.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Figure

ACKNOWLEDGEMENT

I wish to thank my thesis advisor, Professor Dr. S. R. Parker, for his assistance and advice. His guidance has helped me to investigate the material for this thesis. He has taken much of his time to discuss the problems that existed during this investigation.

I. INTRODUCTION

In this age of computers, several excellent computer circuit-analysis programs have been developed which solve the equations describing electrical networks with remarkable accuracy. In order for the computed results to agree with actual circuit performance, the procedure of device modeling must furnish accurate models which are computationally efficient. As stated in a paper published by Design Automation, Inc. [1], "For all its advantages, the computer analysis of a circuit can be no better than the data that are fed into it. Poor models or improperly assigned values yield inaccurate results. In fact, once a circuit-analysis program is running on a computer and the designer has developed the skill to use it, the choice of semiconductor models and procedures for finding numerical values for the models' parameters are the main stumbling blocks to applying the program. The parameters of a real switching transistor or real diode vary as functions of voltage, current and temperature. Modeling is most accurate when the device characteristics are measured under conditions of voltage, current and junction temperature similar to those under which the device is expected to operate." After a model has been chosen, it is essential that the various parameters must be computed easily and accurately. With nonlinear devices, such as diodes and transistors, the model equations must give a mathematical description of the curve so that for a given voltage the current is known or vice versa. For this analysis, the

Ebers-Moll [2] models (modified slightly) for the diode and the transistor are usually employed. Using these models, methods are needed to determine the various parameters necessary for computation using any of the several computer circuit-analysis programs (TRAC, CIRCUS, NET I, NET II, and SCAN). These models cover both small-signal linear and large-signal nonlinear operation. The analysis in this thesis is limited to determining model parameters using steady-state characteristics.

In May, 1965, engineers at North American Aviation, Inc. completed analysis on the least-square-error curve fit for the semiconductor diode model parameters for the SCAN and TRAC computer circuit analysis programs. The modified Ebers-Moll model for the diode was presented and the necessary assumptions were included.

Design Automation, Inc., in June 1967, proposed a modeling procedure for transistors and diodes for computer-aided nonlinear circuit analysis [1]. The models presented were essentially the Ebers-Moll models with additions. The analysis by Design Automations, Inc. was developed for the NET I Network Analysis Program. Several improvements were made to the existing models.

In June 1968, the junction transistor model (modified Ebers-Moll model) was presented by E. Steele for the TRAC computer analysis program [3]. In March 1969 engineers at North American Rockwell Corporation presented, in an internal letter, a proposed method, using least-square estimation, of deriving the various parameters by analyzing the two junctions of the transistor separately.

In this thesis the problem of estimating the parameters of the Ebers-Moll models is considered. Questions investigated are: (1) determination of which regions of a semiconductor device are most useful for estimating each parameter of the model, (2) development of computer techniques of parameter estimation using least-square-error and normalized least square error solutions, and (3) estimation of the accuracy of the estimations. The ultimate question is whether the proposed models give an accurate representation of the actual performance of the semiconductor devices.

In Chapter II the modified Ebers-Moll models for the diode and the transistor are presented. These models allow accurate calculations to be made of the semiconductor device parameters necessary for the computer circuit-analysis programs.

In Chapter III the two methods of least-square-error solution are presented. One method is based upon the usual least-square-error solution and the other is based upon a normalized least-square-error solution as developed by Werther and Parker [4].

Procedures for calculating the semiconductor device parameters are presented for both the diode and the junction transistor. A discussion of the useful regions of parameter determination and the typical values expected from this analysis is included. A method of accumulating measured data and correcting estimated values accordingly in also discussed.

The parameters for typical diodes are calculated in the Appendix, using the two least-square-error methods. A comparison

of the results of these two methods is made and includes calculation of the error after the model parameters have been estimated.

The computer programs used for the solution of the model parameters are given following the Appendix. Recursive equations for both least-square-error methods are used. The FORTRAN IV programming language is used for the computer programs. Most of the subroutines were developed for this analysis, but several were existing subroutines in the Naval Postgraduate School Computer Facility subroutine library.

II. EBERS-MOLL MODELS

Ebers and Moll described the diode model and the junction transistor model, under DC conditions, using the small-signal parameters to relate to large-signal behavior [2]. These models described all regions of operation. Most of the other proposed models are mathematically equivalent and generally less accurate than the Ebers-Moll model [5]. Both the diode and the transistor models are considered below. The Ebers-Moll equations are modified by considering a term called the emission constant M for the diode and M_E and M_C for the transistor. Also linear resistances are added to account for the deviations from the ideal models. The Ebers-Moll models relate the semiconductor device characteristics in terms of several parameters, and generally provide very little insight to the actual physical processes within the devices represented by these models.

A. DIODE P-N JUNCTION MODEL

For an ideal p-n junction diode, the current and voltage have the following relationship

$$I = I_S \left[\exp(qV/kT) - 1\right] \tag{2.1}$$

where I_S is the reverse saturation current

V is the voltage across the terminals

q is the charge on an electron

k is the Boltzman constant

T is the absolute temperature (room temperature-298°K)

kT/q equals 26mV at room temperature.

In practice very few p-n junctions can be described using the ideal diode relationship. In order to account for deviations between the theoretical and actual characteristics the ideal p-n junction diode needs to be modified by placing linear elements in parallel with or in series to the ideal diode. A large parallel resistance called the reverse diode leakage resistance is used to represent the slope of the reverse characteristics, and a small series resistance called the diode ohmic series resistance is used to represent part of the voltage drop across the p-n junction. Also, a modification to the Ebers-Moll model is the quantity M called the diode emission constant, with a value between 1 and 3, which changes the exponential dependence to more exactly describe the actual behavior, because of the non-ideal operation [3]. The modified ideal diode equation becomes

$$I = I_S \left[\exp(qV/MkT) - 1\right] . \tag{2.2}$$

The DC diode circuit is given in Figure 1 using the above model. The various quantities included in Figure 1 are defined as

V_j, I_j - the measured diode terminal voltage and current

R_s - the combined body, lead and contact resistance

R_l - the reverse surface-leakage resistance across the junction

D - the modified ideal diode

I - the ideal diode current given in equation 2.2.

From the measured data of current and voltage, the parameters I_s, R_s, R_l and M are to be determined to describe a specific diode or p-n junction.

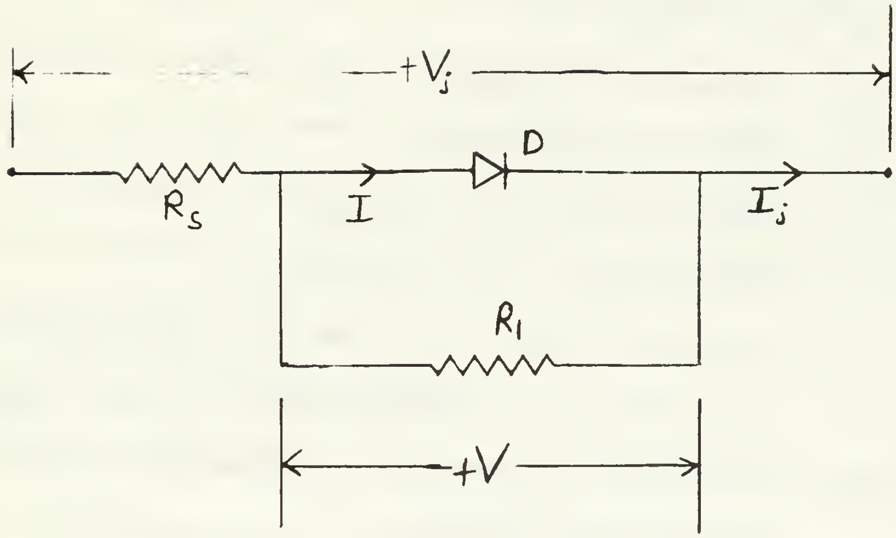


FIGURE 1

DC P-N Diode Circuit Model

B. JUNCTION TRANSISTOR MODEL

For the ideal junction transistor, Ebers and Moll proposed the following current-voltage relationship assuming a pnp transistor [2].

$$I_E = \frac{I_{EO}}{1-\alpha_N \alpha_I}\left[\exp(qV_E/kT)-1\right] - \frac{\alpha_I I_{CO}}{1-\alpha_N \alpha_I}\left[\exp(qV_C/kT)-1\right] \quad (2.3)$$

$$I_C = \frac{-\alpha_N I_{EO}}{1-\alpha_N \alpha_I}\left[\exp(qV_E/kT)-1\right] + \frac{I_{CO}}{1-\alpha_N \alpha_I}\left[\exp(qV_C/kT)-1\right] \quad (2.4)$$

$$\alpha_I I_{CO} = \alpha_N I_{EO} \quad (2.5)$$

where I_E = the emitter current

I_C = the collector current

V_E = the emitter-to-base voltage

V_C = the collector-to-base voltage

$_N$ = the common-base normal-mode DC current gain

$_I$ = the common-base inverted-mode DC current gain

I_{EO} = the saturation current of the emitter junction with zero collector current.

I_{CO} = the saturation current of the collector junction with zero emitter current.

From this the following is true [3]

$$I_{ES} = I_{EO}/(1-\alpha_N \alpha_I) \quad (2.6)$$

$$I_{CS} = I_{CO}/(1-\alpha_N \alpha_I) \quad (2.7)$$

where I_{ES} = the emitter-to-base p-n junction saturation current

I_{CS} = the collector-to-base p-n junction saturation current.

In order to make the exponential dependence more exact, two quantities M_E and M_C are needed similar to the diode exponential emission constant. The quantity M_E is the emission constant for the emitter-base p-n junction and the quantity M_C is the emission constant for the collector-base p-n junction. The ideal transistor equations then become

$$I_E = I_{ES}\left[\exp(qV_E/M_E kT) - 1\right] - \alpha_I I_{CS}\left[\exp(qV_C/M_C kT) - 1\right] \quad (2.8)$$

and

$$I_C = I_{CS}\left[\exp(qV_C/M_C kT) - 1\right] - \alpha_N I_{ES}\left[\exp(qV_E/M_E kT) - 1\right]. \quad (2.9)$$

Linear circuit elements are then added to modify the ideal transistor to an equivalent element to describe the actual relationship, similar to the diode formulation. The transistor equivalent circuit is presented in Figure 2 [1]. The elements of Figure 2 are defined below:

I_B = $I_E + I_C$ the base current

I_2 = the current generator across the collector-base junction

$= I_{CS}\left[\exp(qV_C/M_C kT) - 1\right] - \alpha_N I_{ES}\left[\exp(qV_E/M_E kT) - 1\right]$

I_1 = the current generator across the emitter-base junction

$= I_{ES}\left[\exp(qV_E/M_E kT) - 1\right] - \alpha_I I_{CS}\left[\exp(qV_C/M_C kT) - 1\right]$

R_{BB} = the base spreading, bulk and contact resistance

R_{EE} = the emitter bulk and contact resistance

R_{CC} = the collector bulk and contact resistance

R_E = the emitter-base junction ohmic leakage resistance

R_C = the collector-base junction ohmic leakage resistance

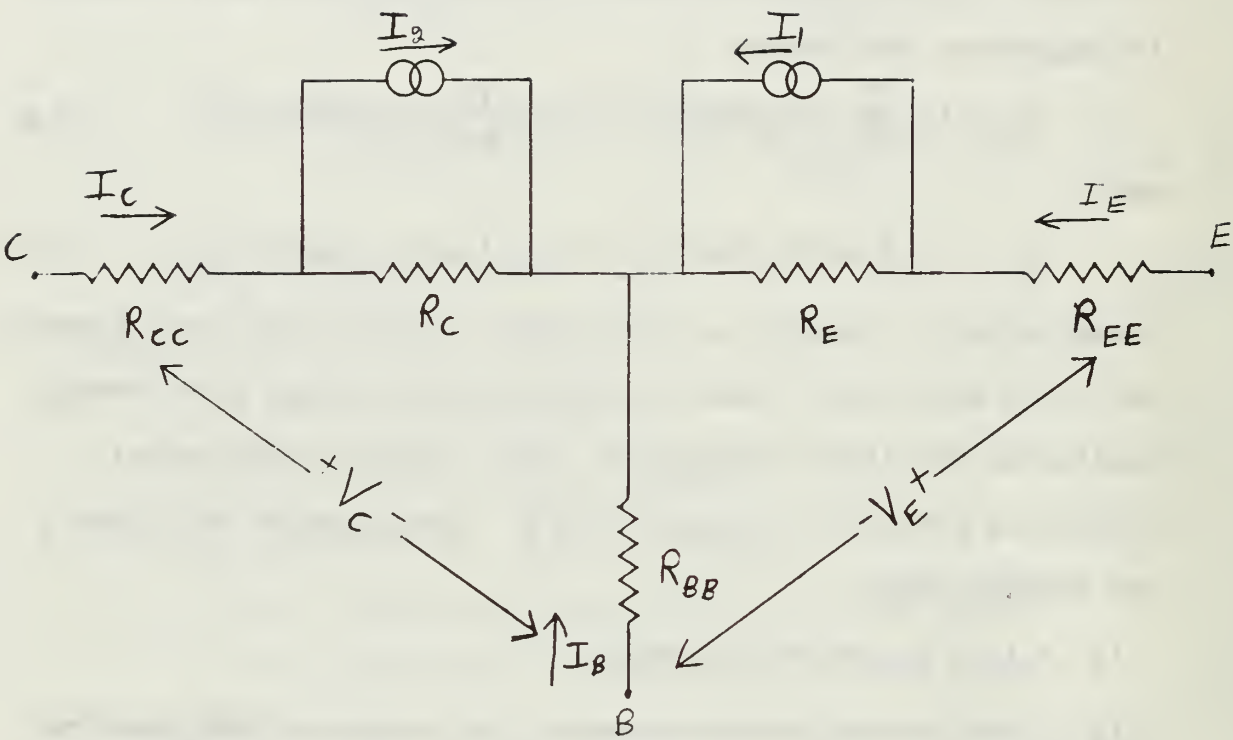


FIGURE 2

DC Junction Transistor Circuit Model

The equivalent circuit for a pnp junction transistor may be used for a npn transistor by changing the polarities of V_C and V_E and the directions of I_1 and I_2.

The specification sheets for transistors usually specify common-emitter DC current gains H_{FE_n} and H_{FE_i} rather than the common-base DC current gains α_N and α_I. The relationship between these α's and H_{FE}'s are

$$\alpha_N = H_{FE_n}/(H_{FE_n}+1) \qquad (2.10)$$

and

$$\alpha_I = H_{FE_i}/(H_{FE_i}+1)\,. \qquad (2.11)$$

A simplified ideal model of the junction transistor is given in Figure 3, where I_1 and I_2 are replaced with diodes and dependent current sources. The resistances R_E and R_C are assumed to be very large and R_{BB}, R_{CC}, and R_{EE} are assumed to be small. For Figure 3 the elements are defined below

$$I_{EF} = I_{ES}\left[\exp(qV_E/M_EkT)-1\right] \qquad (2.12)$$

and

$$I_{CF} = I_{CS}\left[\exp(qV_C/M_CkT)-1\right]. \qquad (2.13)$$

From measured data, the parameters R_{CC}, R_{EE}, R_{BB}, M_E, M_C, I_{ES}, I_{CS}, H_{FE_n}, and H_{FE_i} are to be determined to describe a specific transistor.

The complete model including the AC components for the diode is given in Figure 4. The AC components added to the diode model are

C_t - the junction transition capacitance

and

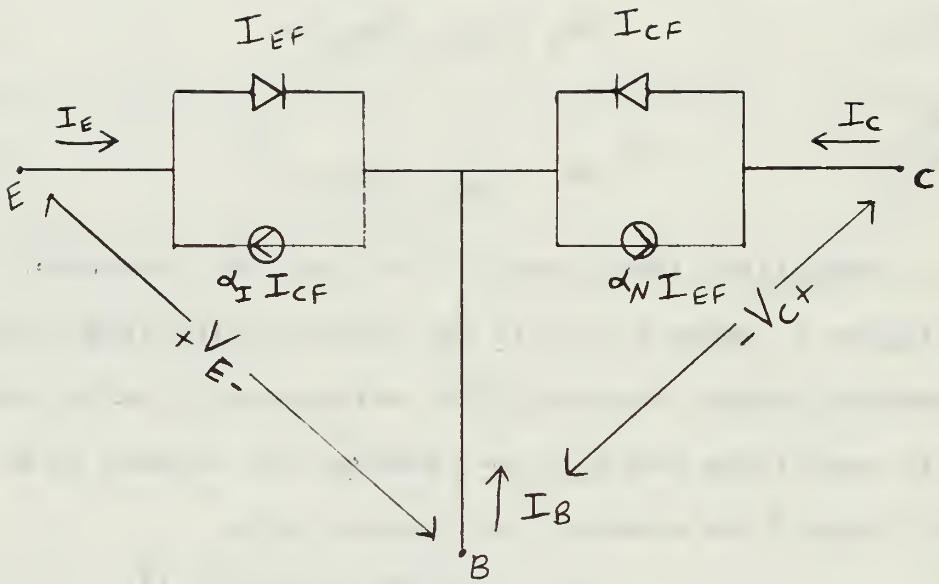


FIGURE 3

Simplified Transistor Model

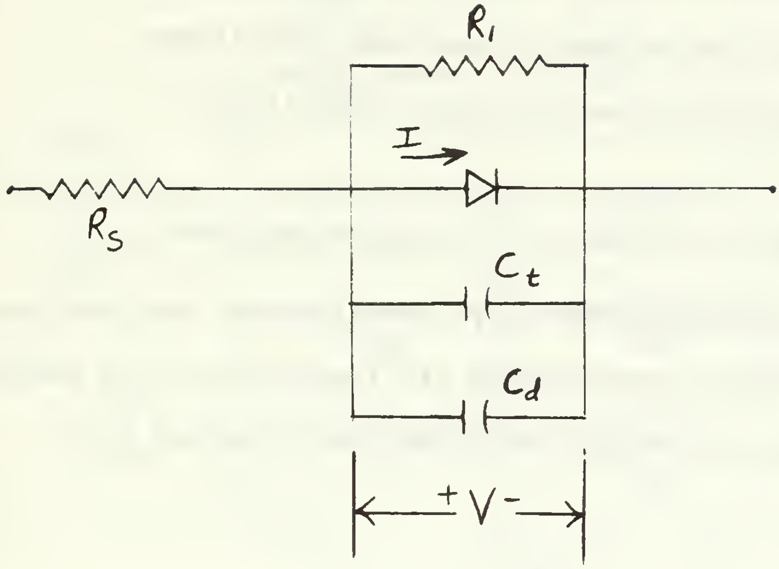


FIGURE 4

Complete Diode Model

C_d - the junction diffusion capacitance.

In Figure 5 the AC components have been added to the transistor model so all operating conditions except the break-down regions for the junctions can be accounted in the model. The added components are

C_{tE} - the emitter-base transition capacitance,

C_{tC} - the collector-base transition capacitance,

C_{dE} - the emitter-base diffusion capacitance,

and

C_{dC} - the collector-base diffusion capacitance.

The transition capacitances are functions of the junction voltages and the diffusion capacitances are functions of the emitted currents from the emitter and from the collector [1].

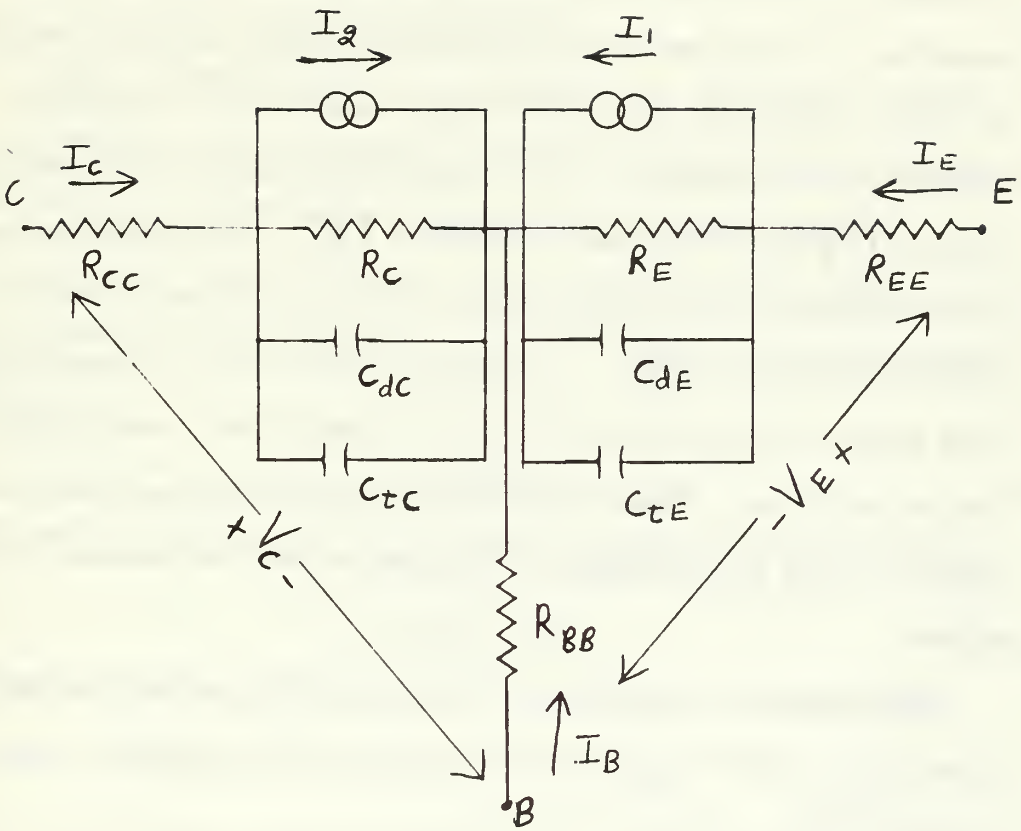


FIGURE 5

Complete Transistor Model

III. LEAST-SQUARE-ERROR SOLUTIONS

The method chosen to compute the various parameters of the models presented is the least-square-error estimation. This solution is for a set of overdetermined simultaneous linear equations. There are two least-square-error methods presented. The first method is the usual least-square-error solution utilizing the concepts of the pseudo inverse as developed by Penrose [6] and presented by Werther [4]. The second method is the normalized least-square-error solution as developed by Werther and Parker [4]. A recursive equation is presented for each method to facilitate the use of additional data points in the calculation. These two methods and the estimations are compared with measurements to determine the usefulness of each method.

The material for the least-square-error estimation and the normalized least-square-error estimation was obtained from Werther [4].

A. LEAST-SQUARE-ERROR SOLUTION

The least-square-error solution is a very common method of solving a set of m simultaneous linear equations in n unknown, m being greater than n. This is the overdetermined case for linear equations which is common when measurements are made to find a small number of unknowns. The least-square-error solution

solves the relationship[1]

$$A\underline{x} = \underline{b} \tag{3.1}$$

where A is the m by n matrix of coefficients,

$\underline{b}$ is the m by 1 vector of constants,

and

$\underline{x}$ is the n by 1 vector of unknowns.

A unique solution for $\underline{x}$ is obtained by using the concept of the Penrose pseudo inverse. This yields the exact solution if it exists, or the least-square-error approximation if the exact solution does not exist. The Penrose pseudo inverse is a n by n matrix and is identified as A^+. The least-square-error solution is

$$\hat{\underline{x}} = A^+\underline{b}. \tag{3.2}$$

If the matrix A has rank n the least-square-error solution is

$$\hat{\underline{x}} = \left[A^TA\right]^{-1} A^T\underline{b} \tag{3.3}$$

where A^T is the matrix transpose of the matrix A and $\left[A^TA\right]^{-1}$ is the matrix inverse of the product of A^T and A. In order for the rank of A to be n, the rows of A must be independent. This solution is easily obtained on the digital computer with available matrix operation subroutines.

For the recursive relationship, the least-square-error solution for the first k points is

$$\hat{\underline{x}}_k = P_kA_k^T\underline{z}_k \tag{3.4}$$

where $\underline{z}_k$ is equal to $\underline{b}_k$ which is the vector of k points,

---

[1]The bar under a lower-case letter represents a column matrix or vector.

A_k is the k by n coefficient matrix,

P_k is the inverse of the matrix $A_k^T A_k$,

and

$\hat{\underline{x}}_k$ is the least-square-error solution.

Now to consider the k+1 equation, $\underline{a}^T$ is a row of new coefficients to the A matrix and $\underline{z}_{k+1}$ is the new data for the vector $\underline{b}_{k+1}$.

Therefore

$$A_{k+1} = \begin{bmatrix} A_k \\ ---- \\ \underline{a}^T \end{bmatrix} \tag{3.5}$$

and

$$\underline{b}_{k+1} = \begin{bmatrix} \underline{z}_k \\ ---- \\ \underline{z}_{k+1} \end{bmatrix}. \tag{3.6}$$

The k+1 least-square-error solution is

$$\hat{\underline{x}}_{k+1} = P_{k+1} A_{k+1}{}^T \underline{z}_{k+1} \tag{3.7}$$

where P_{k+1} is the inverse of the matrix $\left[A_{k+1}{}^T A_{k+1}\right]$.

The resultant recursive equations are

$$\hat{\underline{x}}_{k+1} = \hat{\underline{x}}_k + \frac{P_k \underline{a}}{1+\underline{a}^T P_k \underline{a}} (\underline{z}_{k+1} - \underline{a}^T \underline{x}_k) \tag{3.8}$$

and

$$P_{k+1} = P_k - \frac{P_k \underline{a}\ \underline{a}^T P_k}{1+\underline{a}^T P_k \underline{a}} . \tag{3.9}$$

B. NORMALIZED LEAST-SQUARE-ERROR SOLUTION

The normalized least-square-error solution is a weighted least-square-error solution with weighting factors chosen such

that the solution lies as close as possible to all geometric loci described by equation 3.1.

The matrix of weighting or normalizing factors is a diagonal m by m matrix called W. The factors w_{ii} are given by

$$w_{ii} = \left(\sum_{j=1}^{n} a_{ij}^2\right)^{-\frac{1}{2}}, \quad i=1,2,\ldots,m \tag{3.10}$$

This weighting matrix is multiplied times the matrix A and vector $\underline{b}$ resulting in two new quantities

$$A^* = WA \tag{3.11}$$

and

$$\underline{b}^* = W\underline{b}. \tag{3.12}$$

The normalized least-square-error solution is then

$$\hat{\underline{x}}^* = \left[A^{*T}A^*\right]^{-1} A^{*T}\underline{b}^*. \tag{3.13}$$

The recursive relationships for the normalized least-square-error solution are

$$\hat{\underline{x}}^*_k = \hat{\underline{x}}^*_{k+1} + \frac{P^*_k\ \underline{a}^*}{1+\underline{a}^*P^*_k\ \underline{a}^*}(\underline{z}^*_{k+1} - \underline{a}^*\underline{x}^*_k) \tag{3.14}$$

and

$$P^*_{k+1} = P^*_k - \frac{P^*_k\ \underline{a}^*\underline{a}^{*T}P^*_k}{1+\underline{a}^{*T}P^*_k\ \underline{a}^*}\ . \tag{3.15}$$

Using the fact that

$$\underline{z}^*_{k+1} = (\underline{a}^T\underline{a})^{-\frac{1}{2}}\underline{z}_{k+1} \tag{3.16}$$

and

$$\underline{a}^* = (\underline{a}^T\underline{a})^{-\frac{1}{2}}\underline{a} \tag{3.17}$$

the recursive relationships become

$$\hat{\underline{x}}^*_{k+1} = \hat{\underline{x}}^*_k + \frac{P^*_k\,\underline{a}}{\underline{a}^T\underline{a}+\underline{a}^T P^*_k\,\underline{a}}(z_{k+1} - \underline{a}^T\underline{x}^*_k) \tag{3.18}$$

and

$$P^*_{k+1} = P^*_k - \frac{P^*_k\,\underline{a}\,\underline{a}^T P^*_k}{1+\underline{a}^T P^*_k\,\underline{a}} \quad . \tag{3.19}$$

The recursive relationships for both the least-square-error solution and the normalized least-square-error solution are easily adapted to the digital computer and a sample computer program for each method is found after the Appendix.

IV. DETERMINATION OF SEMICONDUCTOR PARAMETERS

Using the models presented in Chapter II, the model parameters are calculated utilizing the least-square-error method. Parameter calculation is accomplished using the above equations and the equivalent circuits. In order to use the least-square-error methods, the nonlinear equations characteristic of these semiconductor devices must be converted to a set of linear equations, which involves several assumptions. The solution is made on the nonlinear equations where these equations can be assumed to be linear. The error associated with these assumptions is so small it is considered negligible.

These parameters enable the user to employ these models in existing computer circuit-analysis programs and obtain computed results that very accurately represent the actual performance. The data for parameter estimation may be obtained using the measured current and voltage values or values taken from device specification sheets. When specification sheets are used, the values of the voltage should be read to several millivolts in order to obtain accurate results.

Since the data involves experimental error and is statistical in nature, as many data points as possible should be used in the estimation of the semiconductor parameters. Since the measurements are independent, in general, the rank of the matrix A in equation 3.1 is equal to n, the number of parameters. Since the overdetermined case must have the matrix A with a rank of at least n, the solution presented in Chapter III may be used.

A. LINEAR EQUATION FOR DIODE PARAMETER ESTIMATION

The model for the semiconductor diode is presented in Figure 1. In order to determine the values of the parameters, the model equation has to be simplified and made linear to be compatible with the least-square-error methods. Using Figure 1, the following can be stated:

$$I_j = I + V/R_1, \tag{4.1}$$

or

$$I_j = I_s\left[\exp(qV/MkT) - 1\right] + V/R_1. \tag{4.2}$$

The first restriction is that V_j (the measured terminal voltage) be positive ($V_j > 0$). With this restriction, $\exp(qV/MkT) \gg 1$ because even with small positive measured voltages (for example 0.1 volt) and with $q/MkT = 0.025$, the term $\exp(qV/MkT) = 54.99$. This is much greater than 1. Since R_1 is so very large compared to V, $I \gg V/R_1$, and equation 4.2 is simplified to

$$I_j = I_s\exp(qV/MkT) \tag{4.3}$$

which by rearranging becomes

$$V = MkT/q\ \ln(I_j/I_s). \tag{4.4}$$

Referring back to the model

$$V_j = V + I_jR_s, \tag{4.5}$$

and substituting 4.4 into 4.5 results in

$$V_j = MkT/q\ \ln(I_j/I_s) + I_jR_s. \tag{4.6}$$

This equation is simplified by letting $A_0 = MkT/q$ and $A_1 = 1/I_s$, obtaining

$$V_j = A_0 \ln(A_1 I_j) + I_j R_s \tag{4.7}$$

$$V_j = A_0 \ln(A_1) + A_0 \ln(I_j) + I_j R_s \tag{4.8}$$

$$V_j = A_3 + A_0 \ln(I_j) + I_j R_s, \tag{4.9}$$

where $A_3 = A_0 \ln(A_1)$. This approach is currently used by North American Rockwell Corp., among others.

In the above equation, V_j and I_j are the measured forward-voltage and current data points for the diode. The other three quantities are constants to be determined using the least-square-error procedure. For the least-square-error method

$$A = \begin{bmatrix} 1 & \ln(I_1) & I_1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & \ln(I_m) & I_m \end{bmatrix} \tag{4.10}$$

$$\underline{b} = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_m \end{bmatrix} \tag{4.11}$$

and

$$\underline{\hat{x}} = \begin{bmatrix} A_3 \\ A_0 \\ R_s \end{bmatrix} \tag{4.12}$$

where $\underline{\hat{x}}$ is the least-square-error solution. After A_3, A_0 and R_s are determined, the parameters of primary interest are calculated using the following relationships:

$$M = A_0 q/kT, \tag{4.13}$$

$$A_1 = \exp(A_3/A_0), \tag{4.14}$$

and

$$I_s = 1/A_1. \tag{4.15}$$

The three parameters M, I_s and R_s are then calculated and can be used in the circuit-analysis computer programs.

For determination of R_1, (the reverse diode leakage resistance due to the imperfections, surface effects, and thermally generated carriers in the depletion region) the same equation is used as the one for the determination of the other diode parameters. For this case, data points are used from the reverse characteristics of the diode; thus V_j and I_j are both negative. The basic equation is 4.2. By making an assumption that $\exp(qV/MkT) \ll 1$ for V less than -0.2 volts,[2] equation 4.2 is reduced to

$$I_j = -I_s + V/R_1. \tag{4.16}$$

Solving for V,

$$V = (I_j + I_s)R_1 \tag{4.17}$$

and substituting this into equation 4.5 results in

$$V_j = (I_j + I_s)R_1 + I_j R_s. \tag{4.18}$$

Rearranging,

$$V_j = I_j(R_1 + R_s) + I_s R_1. \tag{4.19}$$

---

[2]With V = -0.2 volts and MkT/q = 0.026, $\exp(qV/MkT) = 4.5\times10^{-5}$ and most measured data is less than -0.2 volts.

Setting $(R_1 + R_s) = A_5$ and $I_sR_1 = A_6$ results in

$$V_j = I_jA_5 + A_6. \tag{4.20}$$

This is solved using the least-square-error procedures with

$$A = \begin{bmatrix} I_1 & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ I_m & 1 \end{bmatrix} \tag{4.21}$$

$$\underline{b} = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_m \end{bmatrix} \tag{4.22}$$

$$\underline{x} = \begin{bmatrix} A_5 \\ A_6 \end{bmatrix}. \tag{4.23}$$

In the above procedure V_j and I_j are the measured voltage and current in the reverse direction of the diode. From the results, R_1 is determined after an assumption is made. From the assumption that $R_1 \gg R_s$, R_1 is then equal to A_5 since $A_5 = R_1 + R_s$ and R_s is so small it can be neglected. A value of reverse saturation current, I_s, may be found from this data but this value is not a good value for this current, especially for silicon diodes. For a 1N540 diode, the reverse saturation current is determined from the reverse characteristics along with R_1. The value obtained is then placedas a constant into the equation used to derive the forward parameters. The other two parameters are derived to see if this reverse saturation current resulted in parameters in the useable range. It is found that the value of R_s is -1.46 ohms and the value of M is 3.67. These values are not

realistic numbers to be used in computer circuit-analysis programs. These values are determined in the Appendix. The reverse characteristics of silicon diodes tend to be more difficult to model, since the ideal diode law breaks down completely as a description for the reverse silicon-diode behavior [7]. In most p-n junctions the reverse current is not completely voltage-independent, due to the surface leakage. R_1 is the only reliable parameter derived from the reverse data.

Several regions of the forward characteristics were investigated to discover where the best parameter estimations can be found. The standard deviation was computed to determine the closeness of fit to the measured points of the various regions. The closeness of fit is an indication of the error involved. The least standard deviation was found to be when the data was utilized along as much of the forward curve as possible. A comparison using the standard deviation from data received in different regions along the curve reveals this fact. Using the points corresponding to the large values of the measured voltage, or far out on the curve, the standard deviation is three times greater than the standard deviation found using the complete curve. The deviation found from the data of small values of voltage, or closest to the origin, is also much larger than the least standard deviation. These calculations are included in the Appendix using the 1N540 diode.

The reverse saturation current determined from the forward characteristics was used to help find R_1 from the reverse characteristics by setting I_s a constant in the derivation. This

method reveals that this value of I_s has very little effect on the value of R_1 obtained. A simplified equation for determining R_1 can be made,

$$R_1 = V_j/(I_j - I_s), \tag{4.24}$$

where V_j and I_j are measurements from the reverse characteristics.

The parameters for two diodes, 1N540 and 1N277 are calculated in the Appendix. The standard deviation for each diode using the parameters determined is also given. The standard deviation was found using the measured voltage V_j and the calculated voltage VC_j from the measured current I_j. Equation 4.9 is solved placing the estimations of the parameters in the equation for the constants and using the various values of measured current. The value obtained is called VC_j. The standard deviation is found by the following relationship

$$\text{standard deviation} = \sqrt{\frac{\sum_{j=1}^{N} (VC_j - V_j)^2}{N - 1}} \tag{4.25}$$

where N is the total number of data points used. This value of the standard deviation is used as an error indicator. The method with the largest standard deviation has the largest error using the same data points. For the two diodes used, it was found that the least-square-error method had a smaller standard deviation than the normalized least-square-error method, for both diodes. The difference is quite small and is not a good indicator of which method is best. From using these programs, it appears that the normalized least-square-error method has an

advantage in calculations when the measurements are at relatively great distances from the actual desired curve. Either method estimates parameters that give a curve that has less deviation than that expected due to variations from diode to diode due to manufacturing procedures and techniques.

B. DETERMINATION OF PARAMETERS FOR JUNCTION TRANSISTORS

The model for the junction transistor was presented in Figure 2. The parameters to be determined are I_{ES}, I_{CS}, M_E, M_C, R_{BB}, R_{CC}, R_{EE}, R_E and R_C. In this analysis, the emitter-base and the collector-base junctions are considered to be two separate diodes for these calculations. For the emitter-base p-n junction parameters, measured data are made with the collector terminal open circuited and for the collector-base p-n junction parameters, measured data are made with the emitter terminal open circuited.

Considering the emitter-base junction as a diode, calculations are made on the forward measured data to determine the quantities I_{ES}, M_E, and R_{SE}, the same as I_S, M and R_S were determined for the discrete diode above. The quantity R_{SE} is the combined emitter-base body, lead and contact resistance. The emitter reverse leakage resistance (R_E) is determined using the reverse characteristics, the same as for the diode.

For the collector-base junction considered as a diode, calculations are made on the forward characteristics to determine the quantities I_{CS}, M_C, and R_{SC}, the same as was computed for the diode. The quantity R_{SC} is the combined collector-base body,

lead and contact resistance. The collector reverse leakage resistance (R_C) is determined using the reverse characteristics, the same as for the diode.

To complete the model parameters for the junction transistor an assumption is made that the emitter bulk resistance is zero. The quantity R_{EE} is the emitter bulk resistance. Using this assumption, R_{BB} is calculated from

$$R_{BB} = (H_{FE_n} + 1) R_{SE} \tag{4.26}$$

The collector bulk resistance is determined to be

$$R_{CC} = R_{SC} - R_{BB}/(H_{FE_i} + 1) \tag{4.27}$$

The quantities H_{FE_n} and H_{FE_i} are the common-emitter DC normal and inverted current gains respectively.

The shift of H_{FE_n} can be evaluated from the specification sheets available on the individual transistors. Specification sheets normally give curves of H_{FE_n} versus I_C. An average value is obtained by taking n values of H_{FE_n} from the curve at equally spaced values of collector current over the range and then dividing by n. This average value of H_{FE_n} can then be used in further calculations of parameters. The parameter H_{FE_i} can be measured directly or assumed to be values as shown in Table I.

The common-base α's can be calculated using equations 2.10 and 2.11.

This procedure of determining the model parameters for the junction transistor are currently used by North American Rockwell Corp., among others.

TABLE I

Typical Values of H_{FE_i}

(Reprinted from the Motorola Switching Transistor Handbook.)

Transistor Type	Typical H_{FE_i}
Planar, silicon epitaxial	0.2
Alloy, uniform base	4
Alloy, diffused base	1
Mesa, diffused base	0.4

C. TYPICAL VALUES FOR MODEL PARAMETERS

For the p-n junction diode the values of M, R_S and I_S are usually determined to be in the following ranges. The value of M is between 1.0 amd 3.0. the value of R_S is between 10^{-5} and 10^{-1} kilohms, the value of I_S silicon is from 10^{-12} to 10^{-6}mA and for germanium is from 10^{-8} to 10^{-2}mA. The quantity R_1 has a value between one and ten megohms.

The junction transistor parameters are usually contained in the following ranges: The value of M_E and M_C is between 1.0 and 3.0, the value of R_{EE} and R_{CC} is between 10^{-5} and 10^{-2} kilohms, and R_{BB} is usually between 10^{-4} and 10^{-1} kilohms. R_{EE} has been assumed to be zero in this analysis. The usual range of I_{ES} is 10^{-14} to 10^{-6}mA for silicon and 10^{-8} to 10^{-2}mA for germanium. The value of I_{CS} is usually an order of magnitude larger than I_{ES} . The quantities R_C and R_E are usually larger than one megohm.

These are only typical values, but if a parameter is substantially outside the usual range, the input data and calculations should be checked for error. These values were presented by Design Automation, Inc. [1].

The parameter values for the two diodes calculated in the Appendix conform to the typical values for the model as is shown in Table II.

In Table III a comparison of the values calculated using the least-square-error methods to the values calculated following the procedures given by Design Automation, Inc. [1]. There is very little difference in the parameter values derived by either method. The standard deviation for the least-square-error methods

TABLE II

Calculated Parameters and Standard Deviations

	1N540		1N277	
	Usual	Normalized	Usual	Normalized
M	1.75	1.76	1.07	1.10
I_s	1.65×10^{-10}	1.86×10^{-10}	2.65×10^{-10}	3.72×10^{-10}
R_s	0.13	0.126	82.83	67.9
SD	0.006	0.0062	0.0025	0.0027

TABLE III

A Comparison of Parameters Derived Using Two Methods

	Design Automation	Least Square	Normalized Least Square
M	1.75	1.75	1.76
I_s	1.5×10^{-10}	1.65×10^{-10}	1.86×10^{-10}
R_s	0.132	0.131	0.126
SD	0.0076	0.006	0.0062

is a little less than that for the method of Design Automation, Inc. This indicates that the least-square-error methods are as well suited to these models as other procedures.

D. COLLECTION OF DATA POINTS

The measured values of current and voltage used in the A matrix and the $\underline{b}$ vector for the least-square-error methods need to be evenly distributed from the minimum value of voltage (at least 0.1 volt) to a value of voltage that corresponds to maximum power. If these values are too close together or on only one portion of the curve, the derived parameters turn out to be negative values or values that do not correspond to the usual range. Most of this type of error is due to the inaccuracy of the measurements of the voltage and current. A typical derivation is presented in the Appendix. After the parameters have been initially estimated, the additional points are utilized in the recursive relationships. The additional points may be taken anywhere between the minimum and the maximum values of voltage. The least standard deviation is found utilizing as much of the forward characteristics as possible.

For the reverse characteristics, fewer points are needed and they may be spread more than the forward characteristics. The reverse leakage resitance is a large value and if equation 4.24 is used only one set of points is needed in the calculation. This calculation need not be too accurate since the reverse leakage resitance is of little importance. Values of current and voltage between -0.2 volts and the breakdown voltage are acceptable.

V. CONCLUSION

A good model has been chosen since it has been shown that these models accurately represent electrical performance of the device over a very wide range and the models are computationally efficient in regards to the computer circuit-analysis programs presently in use. It is found that the error by this method is considerably less than the variation in the characteristics due to manufacturing techniques. The region chosen to measure the data for the parameters for the forward characteristics is represented as the points lying between 0.1 volts and the maximum power rating of the device. The measured data best represents the device when the points cover as much of the curve between the above mentioned points as possible. Considering the two methods of parameter approximation, both methods give results that are well within the accuracy expected. The normalized least-square-error solution does more to eliminate the influence of points located farthest from the desired curve.

The standard deviation, which is a measure of the error in parameter estimation, reflects that these models give an accurate representation of the actual performance of the semiconductor devices. The standard deviation from the least-square-error methods is a few millivolts and most measurements could be this inaccurate. As seen on the forward characteristics presented in Figures 6, 7 and 8, the difference between the actual measurements and computed values is almost indistinguishable.

Parameters were not estimated for a transistor, but the method of determining these parameters is straight-forward and should not present any serious difficulty. Semiconductor devices with more p-n junctions than the transistor can be modeled similarly by dealing with each junction separately as a diode.

A comparison of values of parameters from the least-square estimates with other methods of parameter estimation reveals that the least-square-error methods produce parameter values essentially the same as other procedures. The error for the least-square-error methods is slightly less than other procedures.

APPENDIX

Examples of Parameter Estimation

This section is used to give two examples of the use of the procedures presented in this thesis. Two diodes, one the 1N540 and the other the 1N277, were used in this analysis as the two examples. Measured data of current and voltage were taken in the forward and reverse portions of the two devices to be used to determine the device parameters.

The following is the forward data collected for the two devices with the voltage given in volts and the current in milliamperes.

1N540 Voltage	1N540 Current	1N277 Voltage	1N277 Current
0.4	0.002	0.25	0.002
0.48	0.006	0.27	0.004
0.5	0.0092	0.28	0.006
0.52	0.014	0.285	0.008
0.54	0.021	0.29	0.01
0.56	0.033	0.3	0.014
0.58	0.0555	0.31	0.02
0.6	0.086	0.32	0.025
0.64	0.215	0.34	0.049
0.68	0.483	0.36	0.085
0.72	1.15	0.38	0.14
0.76	3.18	0.4	0.222
0.8	9.15	0.42	0.37
0.84	15.5		
0.88	37.8		
0.92	76.0		
0.96	155.0		
1.0	270.0		

The reverse data for the devices are presented with both the current and the voltage as negative quantities and the current is in milliamperes and the voltage in volts.

1N540		1N277	
Voltage	Current	Voltage	Current
9.0	0.01	2.65	0.004
11.2	0.012	5.8	0.008
13.0	0.014	10.0	0.0125
15.5	0.016	11.5	0.014
19.5	0.20	20.0	0.024
22.5	0.24	30.0	0.0355
26.5	0.0265	36.0	0.0425

Using seven selected points, an A matrix is assembled to start the recursive operation with the forward characteristics. These seven selected points are spaced over as much of the curve as possible so as to take into consideration the influence of different parameters on various regions of the curve. Errors are sometimes encountered when two points are used that lie too close to each other.

The seven points chosen for the A matrix and b vector are

1N540		1N277	
Voltage	Current	Voltage	Current
0.58	0.0555	0.25	0.002
0.6	0.086	0.27	0.004
0.68	0.483	0.285	0.008
0.72	1.15	0.3	0.014
0.88	37.8	0.34	0.049
0.96	155.0	0.38	0.14
1.0	270.0	0.42	0.37

where the voltage is in volts and the current is in milliamperes. The natural logarithms of the currents were determined and the information was then run on the computer programs to start the recursive procedures. The solution of the parameters using only the first seven points for the 1N540 are as follows for the least-square-error methods described above.

	Least Square	Normalized Least Square
A_3	1.0249	1.0233
A_0	0.0453	0.0451
R_s	0.1268	0.1324
A_1	6.656×10^{9}	7.1373×10^{9}
I_s	1.502×10^{-10}	1.4019×10^{-10}
M	1.743	1.734

The solutions for the points for 1N277 are

	Least Square	Normalized Least Square
A_3	0.6097	0.6162
A_0	0.0275	0.0281
R_s	78.2493	71.3765
A_1	4.1626×10^{9}	3.3997×10^{9}
I_s	2.4023×10^{-10}	2.941×10^{-10}
M	1.0587	1.0798

The rest of the measured points are used in the recursive equations and result in the final forward parameters as

1N540

	Least Square	Normalized Least Square
A_3	1.0246	1.0261
A_0	0.0455	0.0458
R_s	0.1308	0.1257
A_1	6.0536×10^{9}	5.3908×10^{9}
I_s	1.6519×10^{-10}	1.855×10^{-10}
M	1.7495	1.7611

1N277

	Least Square	Normalized Least Square
A_3	0.6116	0.6236
A_0	0.0277	0.0287
R_s	82.8309	67.8968
A_1	3.7768×10^{9}	2.6885×10^{9}
I_s	2.6477×10^{-10}	3.7195×10^{-10}
M	1.0666	1.1047

The seven points from the reverse characteristics were only evaluated using the normalized least-square-error method since the only parameter attained from the reverse characteristics is

the reverse leakage resistance R_1. The value of R_1 is of little importance and any value in the megohm range will be adequate. The calculated values of R_1 are

	1N540	1N277
R_1	9.65×10^5	8.69×10^5

Values of these parameters were calculated with the procedures of Design Automation, Inc., [1] using the data as listed above. The following values were obtained.

	1N540
M	1.75
R_s	0.132
I_s	1.5×10^{-10}

It is evident that these values are quite similar to those of the least-square-error methods.

The standard deviation using equation 4.25 was calculated for the three sets of parameters calculated by the above methods. Each standard deviation calculated is listed below.

Standard Deviations

	Design Automation	Least Square	Normalized Least Square
1N540	0.0076	0.006	0.0062
1N277		0.0025	0.0027

In checking the various regions of the curve to be used the standard deviation was calculated for the parameters derived from data far out on the curve. The parameters calculated were M equals 1.872, I_s equals 5.686×10^{-10} and R_s equals 0.101. The

standard deviation for these parameters is 0.0184. This indicates that the best parameters are derived from data covering as much of the curve as possible.

The standard deviation is a measure of the error between the calculated and the actual measurements. It is seen that the deviation is usually only a few milliamperes.

Figures 6, 7, 8, 9, 10 and 11 are plots of the forward characteristics from the measured data and also of the calculated parameters for diodes. From these curves it is clear that the solution obtained by the least-square-error methods is very accurate.

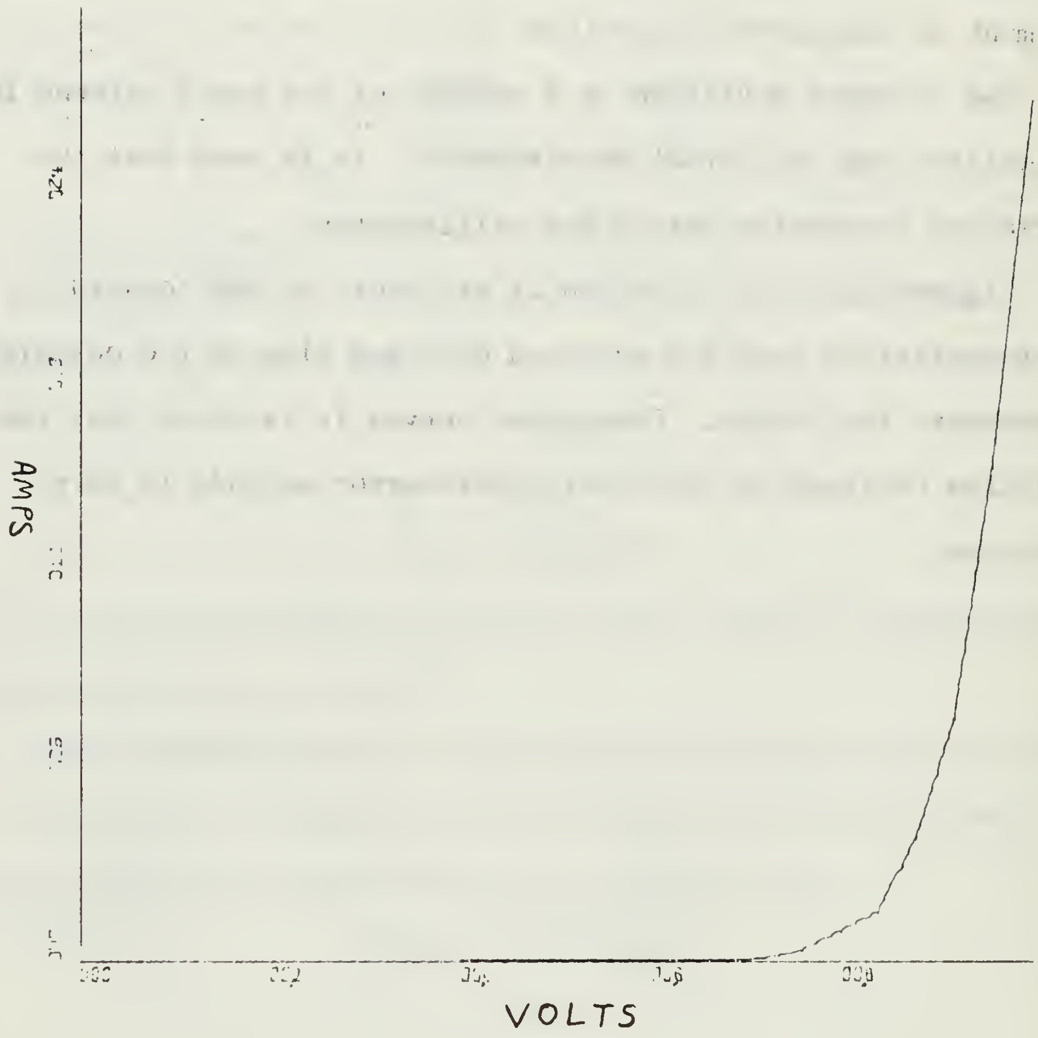
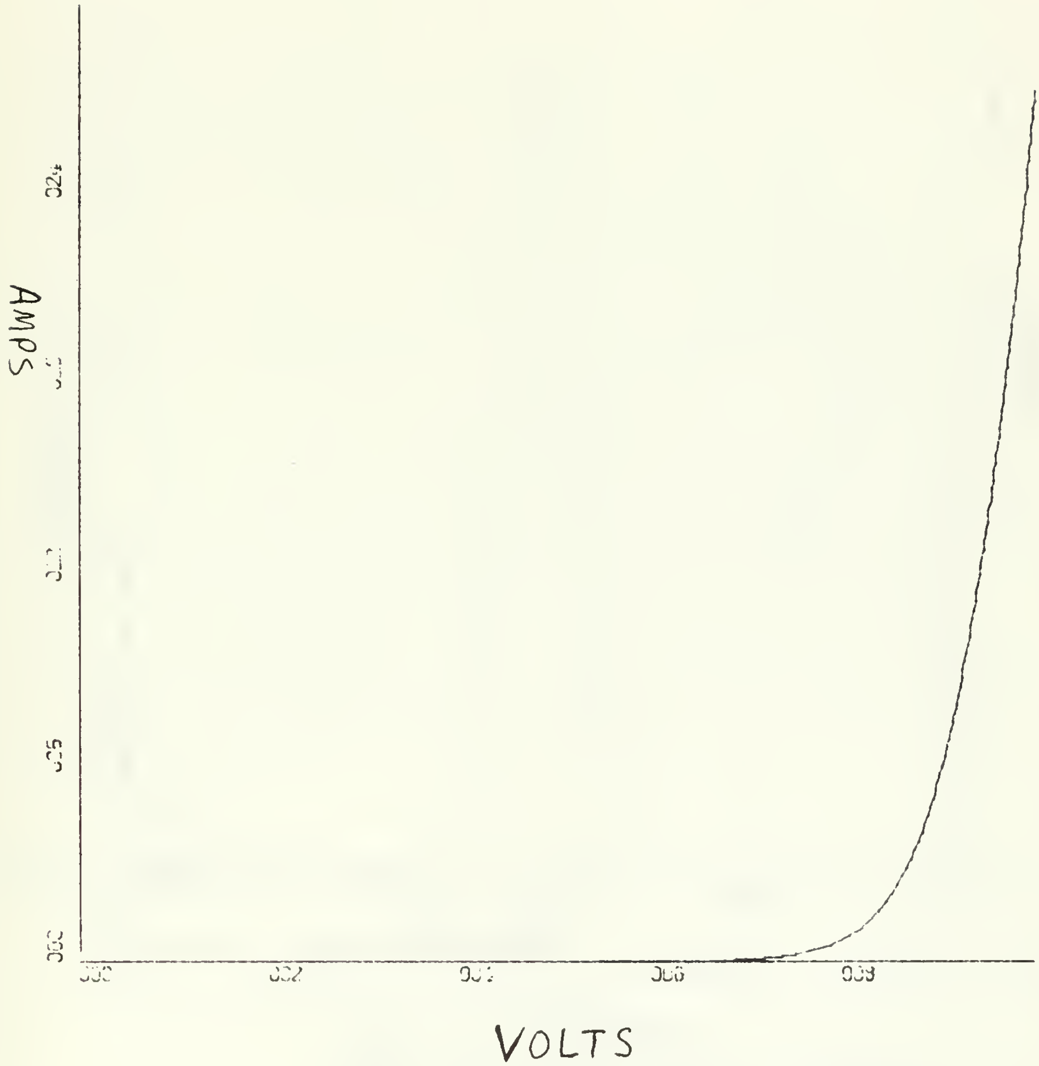


FIGURE 6

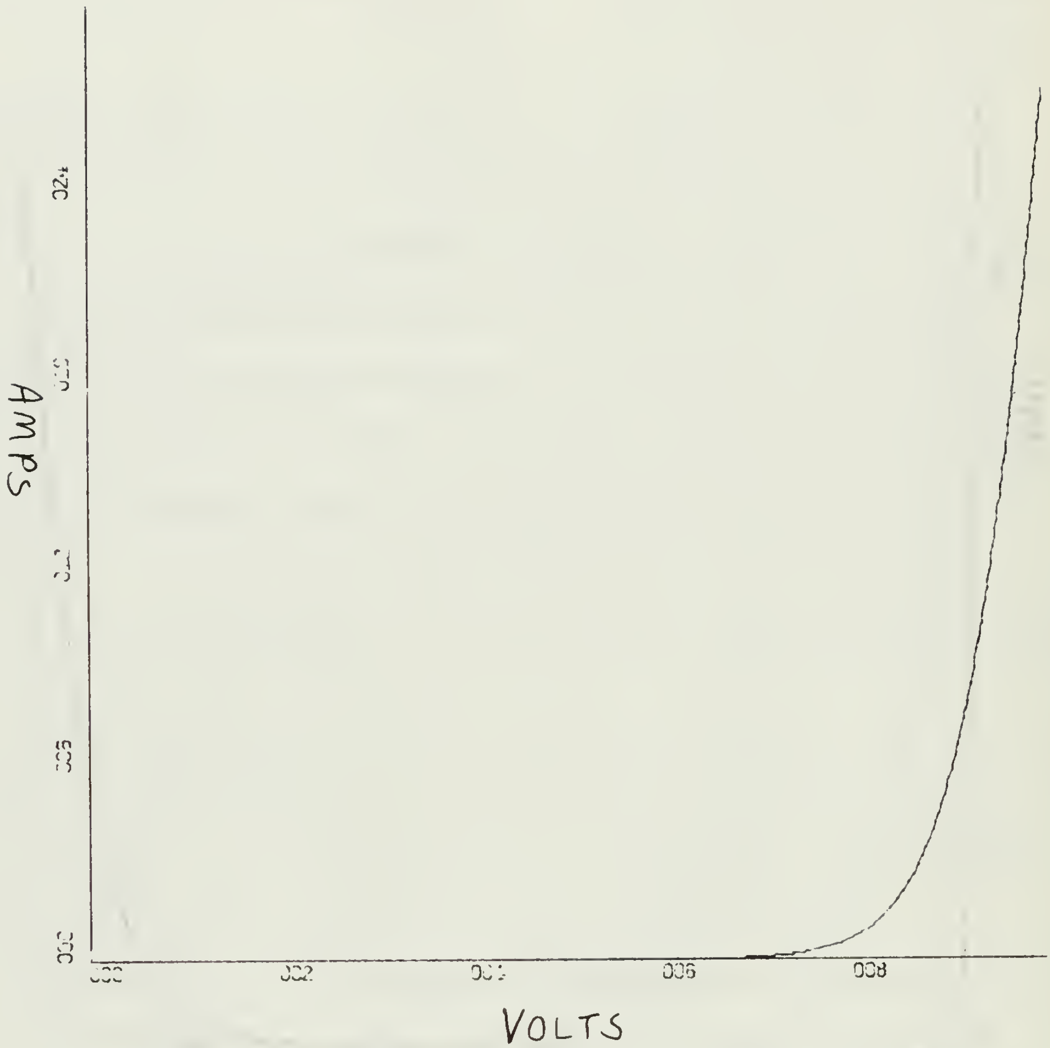
Plot of Measured Points for the 1N540 Diode



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=6.00E-02 UNITS INCH.

FIGURE 7

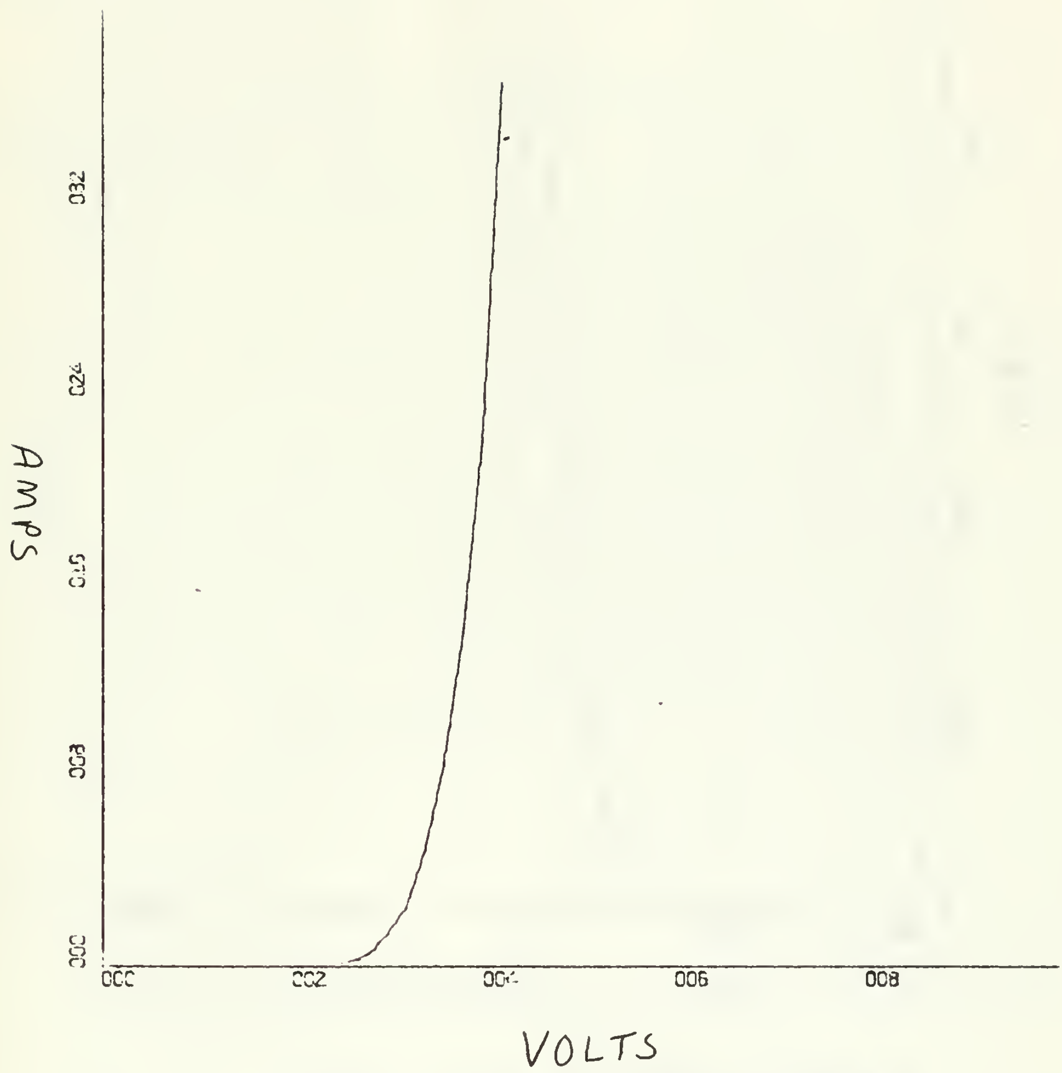
Plot of Calculated Points Using the Least-Square Parameters for the 1N540 Diode



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=6.00E-02 UNITS INCH.

FIGURE 8

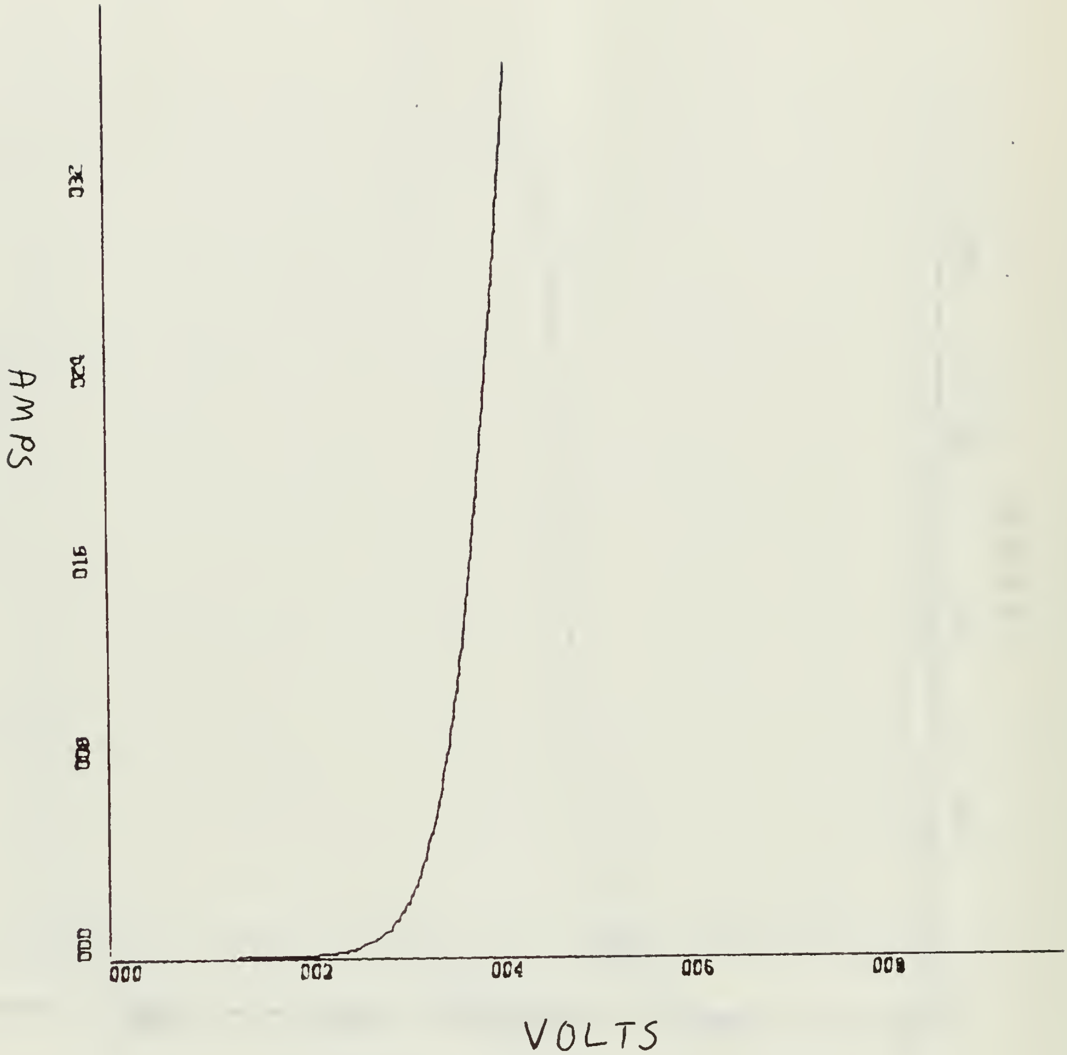
Plot of Calculated Points Using the Normalized Least-Square Parameters for the 1N540 Diode



X-SCALE 2.00E-01 UNITS INCH.
Y-SCALE 8.00E-05 UNITS INCH.

FIGURE 9

Plot of the Measured Points for the 1N277 Diode



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=8.00E-05 UNITS INCH.

FIGURE 10

Plot of Calculated Points Using the Least-Square Parameters for the 1N277 Diode

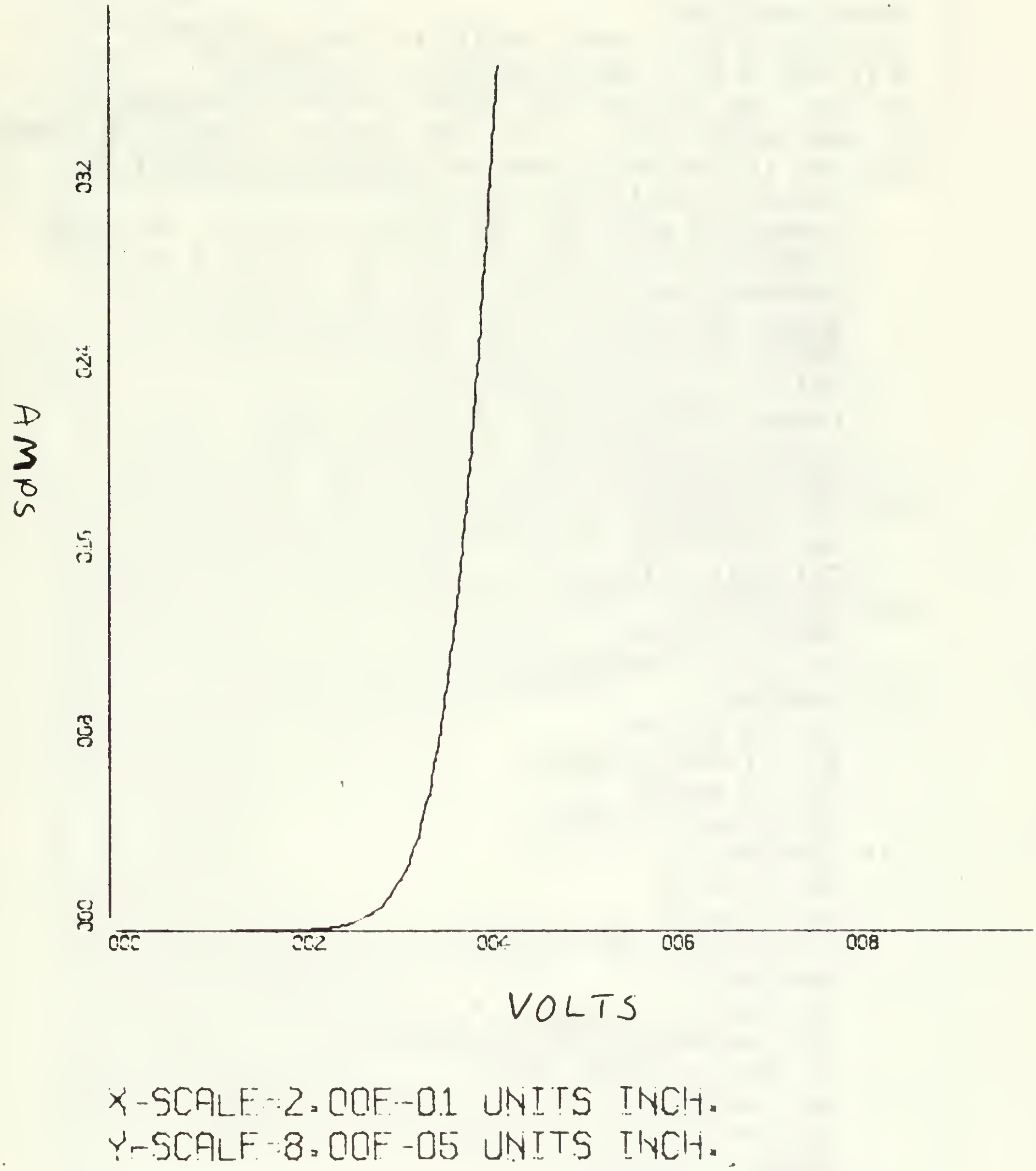


FIGURE 11

Plot of the Calculated Points Using the Normalized Least-Square Parameters for the 1N277 Diode

COMPUTER PROGRAMS

```
C   THIS IS THE MAIN PROGRAM FOR THE NORMALIZED LEAST SQUARE
C   ERROR SOLUTION.
C   A IS THE M BY N INPUT MATRIX OF COEFFICIENTS.
C   B IS THE M BY 1 INPUT VECTOR OF CONSTANTS.
C   RS, SI, AND ETA ARE THE OUTPUT PARAMETERS WHERE RS
C   IS BODY RESISTANCE, SI IS THE REVERSE SATURATION CURRENT
C   AND ETA IS THE VALUE FOR THE EMISSION CONSTANT M..
        IMPLICIT REAL*8 (A-H,O-Z)
        DIMENSION A(10,10), B(10,10), C(10,10), D(10,10)
        DIMENSION W(10,10), T(10), A2(10,10), E(10,10)
        DIMENSION A3(10,10), F(10,10), G(10,10), H(10)
        READ(5,4) M,N
        READ(5,5) ((A(I,J),I=1,M),J=1,N
        READ(5,5) (B(I,1),I=1,M)
      4 FORMAT(2I10)
      5 FORMAT(7F10.0)
        DATA T/10*0.0/
        DO 118 I=1,M
        DO 118 J=1,N
        T(I)=T(I)+A(I,J)**2
    118 CONTINUE
        DO 122 I=1,M
        W(I,I)=1.0/DSQRT(T(I))
    122 CONTINUE
        DO 114 I=1,M
        B(I,1)=B(I,1)*W(I,I)
        DO 114 J=1,N
        A(I,J)=A(I,J)*W(I,I)
    114 CONTINUE
        DO 23 I=1,M
        DO 23 J=1,N
        A2(J,I)=A(I,J)
     23 CONTINUE
        CALL MATMUL(A2,N,M,A,M,N,C,M4,N4,L)
        CALL GAUSS3(N,EPS,C,A3,KER,10)
        IF (KER.EQ.2) GO TO 10
        CALL MATMUL(A3,M4,N4,A2,N,M,D,M5,N5,L2)
        CALL MATMUL(D,M5,N5,B,M,1,G,M8,N8,L3)
        PRINT 8, (G(I,1),I=1,N)
      8 FORMAT(10X,3D14.6)
        DO 303 J=1,10
        READ(5,30) (H(I),I=1,N)
        READ(5,32) Z
     30 FORMAT(3F10.0)
     32 FORMAT(1F10.0)
        CALL WRLSF (A3,H,G,Z,N)
        PRINT 8, (G(I,1),I=1,N)
        A4=G(1,1)
```

```
      A0=G(2,1)
      RS=G(3,1)
      A1=DEXP(A4/A0)
      PRINT 33, A4,A0,RS,A1
   33 FORMAT(10X,4D14.6)
      SI=1.0/A1
      ETA=A0*38.46
      PRINT 8, SI,ETA
  303 CONTINUE
      GO TO 300
   10 WRITE(6,12)
   12 FORMAT(10X,8HSINGULAR)
  300 STOP
      END


C   THIS IS THE MAIN PROGRAM FOR THE LEAST SQUARE ERROR
C   SOLUTION.
C   A IS THE M BY N INPUT MATRIX OF COEFFICIENTS.
C   B IS THE M BY 1 INPUT VECTOR OF CONSTANTS.
C   RS, SI, AND ETA ARE THE OUTPUT PARAMETERS WHERE RS
C   IS BODY RESISTANCE, SI IS THE REVERSE SATURATION CURRENT
C   AND ETA IS THE VALUE FOR THE EMISSION CONSTANT M.
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(10,10), B(10,10), C(10,10), D(10,10)
      DIMENSION A2(10,10), A3(10,10), G(10,10), H(10)
      READ(5,4) M,N
      READ (5,5) ((A(I,J),I=1,M),J=1,N)
      READ(5,5) (B(I,1),I=1,M)
    4 FORMAT(2I10)
    5 FORMAT(7F10.0)
      DO 23 I=1,M
      DO 23 J=1,N
      A2(J,I)=A(I,J)
   23 CONTINUE
      CALL MATMUL(A2,N,M,A,M,N,C,M4,N4,L)
      CALL GAUSS3(N,EPS,C,A3,KER,10)
      IF (KER.EQ.2) GO TO 10
      CALL MATMUL(A3,M4,N4,A2,N,M,D,M5,N5,L2)
      CALL MATMUL(D,M5,N5,B,M,1,G,M8,N8,L3)
      PRINT 8, (G(I,1),I=1,N)
    8 FORMAT(10X,3D14.6)
      DO 303 J=1,10
      READ(5,30) (H(I),I=1,N)
      READ(5,32) Z
   30 FORMAT(3F10.0)
   32 FORMAT(1F10.0)
      CALL TRLSF(A3,H,G,Z,N)
      PRINT 8, (G(I,1),I=1,N)
      A4=G(1,1)
      A0=G(2,1)
      RS=G(3,1)
      A1=DEXP(A4/A0)
```

```
      PRINT 33, A4,A0,RS,A1
   33 FORMAT(10X,4D14.6)
      SI=1.0/A1
      ETA=A0*38.46
      PRINT 8, SI,ETA
  303 CONTINUE
      GO TO 300
   10 WRITE(6,12)
   12 FORMAT(10X,8HSINGULAR)
  300 STOP
      END


      SUBROUTINE WRLSF(A3,H,G,Z,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A3(10,1),H(1), Q(10),V2(10), G(10,1)
      DIMENSION Q2(10,10),Q3(10,10), G1(10)
      CALL MATVEC(A3,M,M,H,Q)
      CALL VECMUL(H,H,M,S)
      CALL VECMUL(H,Q,M,V)
      V1=S+V
      DO 122 I=1,M
  122 V2(I)=Q(I)/V1
      DO 132 I-1,M
      G1(I)=G(I,1)
  132 CONTINUE
      CALL VECMUL(H,G1,M,Z2)
      Z3=Z-Z2
      DO 124 I=1,M
  124 V2(I)=V2(I)*Z3
      DO 126 I=1,M
  126 G1(I)=G1(I)+V2(I)
      DO 134 I=1,M
      G(I,1)=G1(I)
  134 CONTINUE
      CALL MULVEC(M,Q,H,Q2)
      CALL MATMUL(Q2,M,M,A3,M,M,Q3,M,M,L9)
      Q4=1.0D0+V
      DO 128 I=1,M
      DO 128 J=1,M
  128 Q2(I,J) = Q3(I,J)/Q4
      DO 130 I=1,M
      DO 130 J=1,M
      A3(I,J)=A3(I,J)-Q2(I,J)
  130 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRLSF(A3,H,G,Z,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A3(10,1),H(1), Q(10), V2(10), G(10,1)
      DIMENSION Q2(10,10), Q3(10,10), G1(10)
      CALL MATVEC(A3,M,M,H,Q)
      CALL VECMUL(H,Q,M,V)
      V1=1.0+V
      DO 122 I=1,M
  122 V2(I)=Q(I)/V1
      DO 132 I=1,M
      G1(I)=G(I,1)
  132 CONTINUE
      CALL VECMUL(H,G1,M,Z2)
      Z3=Z-Z2
      DO 124 I=1,M
  124 V2(I)=V2(I)*Z3
      DO 126 I=1,M
  126 G1(I)=G1(I)+V2(I)
      DO 134 I=1,M
      G(I,1)=G1(I)
  134 CONTINUE
      CALL MULVEC(M,Q,H,Q2)
      CALL MATMUL(Q2,M,M,A3,M,M,Q3,M,M,L9)
      DO 128 I=1,M
      DO 128 J=1,M
  128 Q2(I,J)=Q3(I,J)/V1
      DO 130 I=1,M
      DO 130 J-1,M
      A3(I,J)=A3(I,J)-Q2(I,J)
  130 CONTINUE
      RETURN
      END

      SUBROUTINE VECMUL(X,Y,M,SUM)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(1),Y(1)
      SUM = SUM-SUM
      DO 5 I=1,M
      SUM=SUM+X(I)*Y(I)
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MULVEC(M,X,Y,XY)
      IMPLICIT REAL*8 (A-H,0-Z)
      DIMENSION X(1),Y(1),XY(10,1)
      DO 6 I=1,M
      DO 5 J=1,M
    5 XY(I,J)=X(I)*Y(J)
    6 CONTINUE
      RETURN
      END


      SUBROUTINE MATVEC(A,M,N,V,P)
      IMPLICIT REAL*8 (A-H,0-Z)
      DIMENSION A(10,1),V(1),P(1)
      DO 6 I=1,M
      P(I)=P(I)-P(I)
      DO 6 J=1,N
      P(I)=P(I)+A(I,J)*V(J)
    6 CONTINUE
      RETURN
      END


      SUBROUTINE MATMUL(A,MA,NA,B,MB,NB,C,MC,NC,L)
      IMPLICIT REAL8* (A-H,0-Z)
      DIMENSION A(10,1),B(10,1),C(10,1)
      L=0
      IF (NA.NE.MB) RETURN
      L=1
      MC=MA
      NC=NB
      DO 12 J=1,NC
      DO 12 I=1,MC
      C(I,J)=0.0
      DO 12 K=1,NA
   12 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

The other subroutine GAUSS3 is a subroutine in the Naval Postgraduate School Computer Facility subroutine library for calculating the inverse of a matrix if such an inverse exists.

These programs are not the most computationally efficient but are adequate for this calculation. There are several improvements that can be made to these programs expecially in regard to storage required.

BIBLIOGRAPHY

1. Design Automation, Inc., Application Note 4.2, Modeling Transistors and Diodes for Computer-aided Nonlinear Circuit Analysis, printed in Electronic Design, 7 June 1967, pp. 54-59, 21 June 1967, pp. 60-66, 5 July 1967, pp. 80-83.

2. Ebers, J. J., and Moll, J. L., "Large Signal Behavior of Junction Transistors, "Proceedings of the IRE, v. 43, pp. 1762-1764, December 1954.

3. Steele, E. L., Nonlinear Device Models for the TRAC Computer Code, paper presented at the Second Asilomar Conference on Circuits and Systems, Pacific Grove, Calif., IEEE number 68C64-ASIL, pp. 101-107, November 1968.

4. Werther, M. H., Recursive Algorithms for the Best Approximate Solution of Linear Equations with Applications to System Identification and State Estimation, Ph.D., Thesis, Naval Postgraduate School, Monterey, Calif., September 1969.

5. Hamilton, D. J., Lindholm, F. A., and Narud, J. A., "Comparison of Large Signal Models for Junction Transistors," Proceedings of the IEEE, v. LII, pp. 239-248, March 1964.

6. Penrose, R., "On Best Approximate Solutions of Linear Matrix Equations, "Cambridge Philosophical Society Proceedings, v.52, pp. 17-19, January 1956.

7. Howard, W. G., "Device Modeling for Computer-aided Circuit Analysis," paper presented at the Western Electronic Show and Convention, session 23, Computer-aided Circuit Design and Testing, August 19-22, 1969.

8. Pritchard, R. L., "Advances in the Understanding of the P-N Junction Triode, "Proceedings of the IRE, v. 46, pp. 1130-1139, June 1958.

9. Daniel, M. E., "Development of Mathematical Models of Semiconductor Devices for Computer-aided Circuit Analysis, "Proceedings of the IEEE, v. 55, pp. 1913-1920, November 1967.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Naval Ships Systems Command (Code 2052) Department of the Navy Washington, D. C. 20360	1
4.	Naval Electronic Systems Command Department of the Navy Washington, D. C. 20360	1
5.	Professor S. R. Parker Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	2
6.	LT William A. Crumly R. R. 1 Letts, Iowa 52754	2

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Parameter Estimation for the Mathematical Modeling of Semiconductors Using the Ebers-Moll Equations

4. DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*

Master's Thesis; December 1969

5. AUTHOR(S) *(First name, middle initial, last name)*

William A. Crumly

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1969	62	9

8a. CONTRACT OR GRANT NO. b. PROJECT NO c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

An accurate method of parameter estimation for the mathematical modeling of semiconductors using the Ebers-Moll equations is presented. Its usefulness is apparent in estimating parameters to be used in computer circuit-analysis programs that have been developed. The Ebers-Moll models were modified to better represent the actual characteristics. The least-square-error methods presented for estimating the parameters are easy to program to the digital computer and result in parameters that are quite accurate in describing the actual characteristics. This procedure yields solutions for parameters that are quite helpful to the engineer in solving electrical circuits involving p-n diodes and junction transistors.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Estimation Semiconductor Parameters Ebers-Moll Models Least-Square-Error Curve Fit						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification

A-31409